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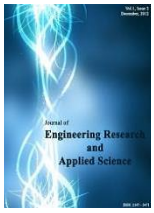

Use of Butterworth Filters for Real Time RMS Value Measurement

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Abstract

RMS (Root Mean Square) value is the most common value that is used quantitative evaluation of currents and voltages in electrical energy systems. The most commonly used method to obtain this value is based on numerical integration of the signal. Methods based on numerical integration have disadvantages such as high computational load, requiring precise determination of frequency and being sensitive to noise. As an alternative method, methods based on filtering for RMS calculation do not have the disadvantages mentioned. In this study, RMS value calculation using digital low-pass Butterworth filter is introduced and investigated. For investigation, Butterworth filters with various orders and cut-off frequencies are simulated and compared in MATLAB. The response of this filtering-based method to sudden changes in the amplitude and frequency of the input signal is also simulated and examined. The results clearly demonstrate that the method is suitable for real-time RMS value computation, with its low computational load and robustness to frequency changes.

Key Words: *Butterworth Filters, Real Time Measurement, RMS Measurement, Low-Pass-Filter.*

1. Introduction

One of the most important values of voltage and current, which are the basic quantities of electrical power systems, is the Root Mean Square (RMS) value. The RMS value is the most common value used to monitor and characterize power system parameters [1]. This value is the DC equivalent of these quantities, which generally have an AC waveform that dissipate the same power in a resistive load. To measure the RMS values of AC voltages and currents, which are actually analog signals; these signals are first converted to digital data using ADCs and then sent into the microprocessor environment. RMS values are calculated from these digital values using some calculation algorithms used in the digital environment of microprocessors [2]. The major numerical methods used in calculations are: Fourier-based methods [3], Wavelet-based methods [4, 5], methods based on numerical integration [6, 7], and methods based on filtering [8]. Perhaps the most widely used of these methods is the method based on digital integration, which uses the average of the squared values of the input signal sampled over one period for the RMS measurement. However, the frequency alteration that often occurs in real current and voltage signals requires a change in the sampling window width. Furthermore, high-frequency harmonics in the input signal can cause calculation errors. To reduce these errors, the sampling frequency is usually increased. Another disadvantage of these methods is that they require a large amount of mathematical processing [9].

In this work, the filtering method that proposed as a solution to these drawbacks is investigated. In this filtering method, the square of the signal to be measured in RMS is subjected to a digital low-pass filter. Here, a Butterworth filter is considered as the low-pass filter, and the RMS value is obtained using a standard IIR filter, the Butterworth filter. Unlike FIR filters, IIR filters, due to their low order, can achieve results by a small number of calculations. However, a poorly designed IIR filter is highly likely to experience instability. As a precaution, IIR filters with standard structures can be used to prevent instability. Low-pass Butterworth filters also offer the advantages of a smooth transition-band amplitude response and a nearly zero stop-band amplitude response.

The main advantages of the method used are that it requires few mathematical operations, is suitable for real-time measurements, and that frequency uncertainty or changes do not affect the results. The following sections of the article first explain how to calculate the RMS value using a filter. The design steps for first-, second-, and third-order Butterworth

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filters are also explained. In the next section, the method is investigated through simulation studies. The analysis is conducted by comparing the performance of filters designed using various filter orders and cut-off frequencies. The final section presents the results of the study and offers recommendations.

2. Method

In electric energy systems, the measurement of current and voltage as fundamental quantities may be necessary for various reasons. Perhaps the most important measurements is the RMS value, also known as the effective value. The RMS value is the DC equivalent of an AC current or voltage in terms of its capacity to do work. If the voltage or current signal exhibits a pure sinusoidal change, we can obtain the RMS value simply by dividing the peak value of the signal by $\sqrt{2}$. However, almost all real voltage or current signals are not pure sinusoidal waves; they also contain harmonics at various frequencies. In this case, to obtain the RMS value of the signal digitally, it is necessary to integrate signal over one fundamental-period (T). Mathematically, this can expressed as in (1)

$$U = \sqrt{\frac{1}{T}\int_{t}^{t+T} u(t)^2 dt} \qquad (1)$$

Here, *U* denotes the RMS value of the voltage *u(t)*. This computation is executed in the digital environment of microprocessor by using samples u_k as in the following equation.

$$U = \sqrt{\frac{1}{N}\sum_{k=0}^{N-1} {u_k}^2} \qquad (2)$$

Denoting main period and sampling period by T and T_s respectively, the variable N in the last equation must provide the following equation.

$$T = NT_s \qquad (3)$$

In practise, the value of N must be at least about 50-100 for precise measurement. This means that the RMS calculations done by (2) would have quite computational load. As a solution to this problem, there are some studies in the literature suggesting the use of digital filters in RMS value measurements. When equation (2) is examined carefully, it can be seen that the expression in the root is an average value calculation process. That is, the average of the squares of the voltage samples is computed. This is equivalent to obtaining the DC component of the signal (the square of the voltage). By considering the frequency of DC component of a signal as zero, it can be filtered from the total signal by a well-designed low-pass filter. Therefore, the RMS value computation can be achieved as given in Fig. 1 by performing the averaging process.

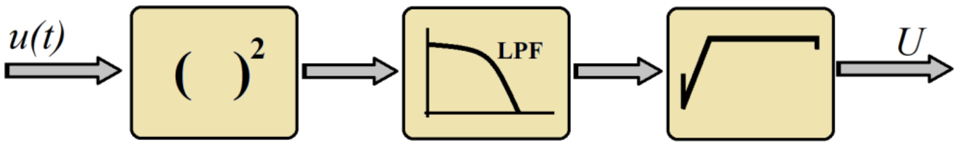

Figure 1. Block scheme of RMS value computation using digital filtering

Therefore, the question arises as to which filter type to use for digital filtering. Digital filters are generally divided into two main classes: finite impulse response filters (FIR filters) and infinite impulse response filters (IIR filters). FIR filters, despite their highly stable operation, have a significantly higher computational burden than IIR filters. In contrast, IIR filters, despite their risk of becoming unstable, have a significantly lower computational burden. It is possible to obtain IIR filters with stable output using some standard filter design methods. Butterworth, Chebyshev Type-I, Chebyshev Type-II, and Elliptic filters are some of the filters with standardized design.

In this study, the use of Butterworth filters is recommended for RMS value measurement due to their stability and low computational burden. The filter's performance is examined in terms of filter order and cut-off frequency. The design of first-, second-, and third-order digital low-pass Butterworth filters is explained below, and the simulation of digital filters designed using the explained method is presented in the next section.

2.1. First Order Low-Pass Butterworth Filter Design

Transfer function of first order low pass Butterworth filter generally is in the form of the following.

$$H(s) = \frac{\omega_c}{s + \omega_c} \tag{4}$$

Here, ω_c is the angular cut-off frequency and is defined by the following equation where f_c denotes the cut-off frequency of the filter.

$$\omega_c = 2\pi f_c \tag{5}$$

In order to convert analogue transfer function to its digital equivalent, there are a few methods. Amongst them, bilinear transform is the most common method. Since the Bilinear Transform is not linear, there is a difference between the analog and digital cut-off frequencies. Therefore, during the conversion, the cut-off frequency is subjected to a pre-processing called pre-warping. This process executed using following equation.

$$\Omega_c = \frac{2}{T_s} tan\left(\frac{\omega_c T_s}{2}\right) \tag{6}$$

In the last equation, T_s denotes the sampling period of the signal. After pre-warping, the transfer function is converted to the following form.

$$H(s) = \frac{\Omega_c}{s + \Omega_c} \tag{7}$$

In the next step, the conversion of the transfer function from analog to digital form is performed by replacing the variable s with its coequal given in the (8).

$$s = \frac{2}{T_s}\frac{1 - z^{-1}}{1 + z^{-1}} \tag{8}$$

Thus, the normalized transfer function of the filter in discrete form is been obtained.

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} \tag{9}$$

Here, the coefficients are given in the following equations.

$$b_0 = b_1 = \frac{\Omega_c T_s}{2 + \Omega_c T_s} \tag{10}$$

$$a_1 = \frac{2 - \Omega_c T_s}{2 + \Omega_c T_s} \tag{11}$$

The transfer function of a digital filter can also be converted into difference equations. Defining input variable as x and output variable as y, the difference equation for this filter can be written as in (12)

$$y(k) = b_0 x(k) + b_1 x(k-1) - a_1 y(k-1) \tag{12}$$

2.2. Second Order Low-Pass Butterworth Filter Design

Transfer function of first order low pass Butterworth filter generally is in the form of the following.

$$H(s) = \frac{{\omega_c}^2}{s^2 + \sqrt{2}\omega_c s + {\omega_c}^2} \tag{13}$$

Before converting analog filter to its digital equivalent, if pre-warping applied to this transfer function the following equation is obtained.

$$H(s) = \frac{{\Omega_c}^2}{s^2 + \sqrt{2}\Omega_c s + {\Omega_c}^2} \tag{14}$$

In the next step, the conversion of the transfer function from analog to digital form is performed by replacing the variable s with its bilinear coequal and results in normalized transfer function as (15).

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{15}$$

By converting the transfer function of this filter into difference equations, the relationship between the input and output is obtained as (16).

$$y(k) = b_0 x(k) + b_1 x(k-1) + b_2 x(k-2) - a_1 y(k-1) - a_2 y(k-2) \tag{16}$$

2.3. Third Order Low-Pass Butterworth Filter Design

Transfer function of first order low pass Butterworth filter generally is in the form of the following.

$$H(s) = \frac{{\omega_c}^3}{s^3 + 2\omega_c s^2 + 2{\omega_c}^2 s + {\omega_c}^3} \tag{17}$$

By applying pre-warping, this transfer function is been converted to following form.

$$H(s) = \frac{{\Omega_c}^3}{s^3 + 2\Omega_c s^2 + 2{\Omega_c}^2 s + {\Omega_c}^3} \qquad (18)$$

In the next step, if this transfer function is converted from analog to digital form using Bilinear transformation, the normalized transfer function is obtained in the form of (19).

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + +b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}} \qquad (19)$$

By converting the transfer function of this filter into difference equations, the relationship between the input and output is obtained as (20).

$$y(k) = b_0 x(k) + b_1 x(k-1) + b_2 x(k-2) + b_3 x(k-3) - a_1 y(k-1) - a_2 y(k-2) - a_3 y(k-3) \qquad (20)$$

3. Simulations

The presented method in this article has been analysed by simulations executed in MATLAB [10] environment. The signals of interest are voltage and current, and sample voltage and current signals are formed as follows.

$$u(t) = 5 + 220\sqrt{2}\sin\left(100\pi t + \frac{\pi}{3}\right) + 20\sqrt{2}\sin\left(300\pi t + \frac{\pi}{5}\right) + 10\sqrt{2}\sin\left(500\pi t + \frac{\pi}{4}\right) \qquad (21)$$

$$i(t) = 15 + 20\sqrt{2}\sin\left(100\pi t + \frac{\pi}{2}\right) + 10\sqrt{2}\sin\left(300\pi t + \frac{\pi}{3}\right) + 5\sqrt{2}\sin\left(500\pi t + \frac{\pi}{4}\right) \qquad (22)$$

Real RMS values of these signals can be computed as below.

$$U = \sqrt{5^2 + 220^2 + 20^2 + 10^2} = 221.19\ Volt \qquad (23)$$

$$I = \sqrt{15^2 + 20^2 + 10^2 + 5^2} = 27.3861\ Amper \qquad (24)$$

Waveforms of these signals are also given in Fig. 2.

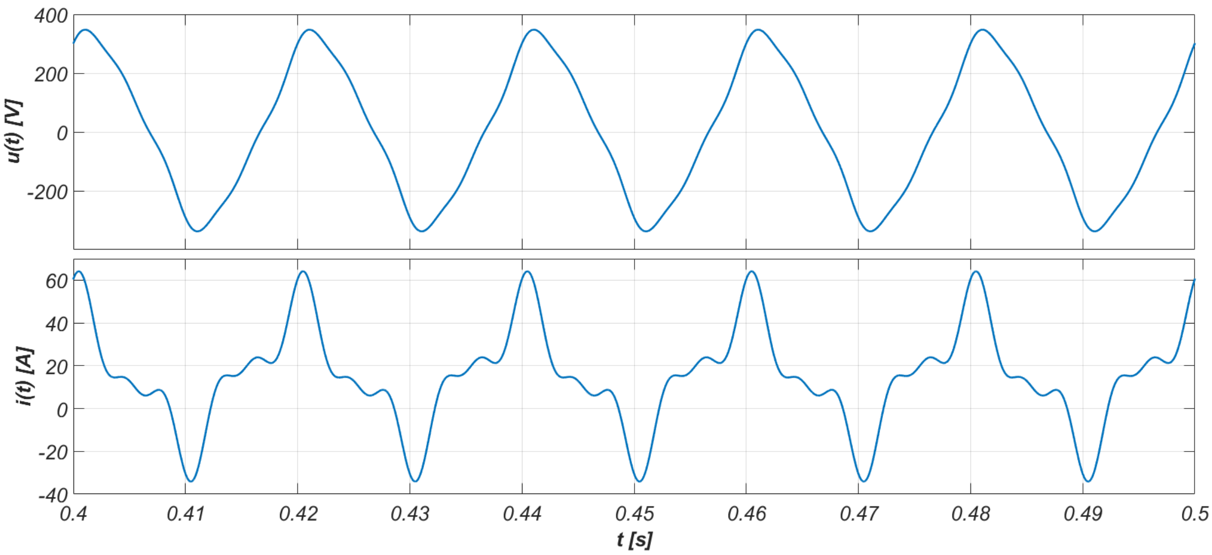

Figure 2. Waveform of voltage and current signals

Considering that the voltage and current are sampled with a sampling frequency of 20 kHz, digital low-pass Butterworth filters are designed as first, second and third order, respectively, and for four different cut-off frequencies (1, 2.5, 5 and 10 Hz) using the design method described above. The transfer functions of the resulting digital Butterworth filters are given in Table 1.

Table 1. Transfer Functions of Designed Digital Butterworth Filters

	1st order	2nd order
f_c=1 Hz	$H(z)=\frac{0.1571x10^{-3}+0.1571x10^{-3}z^{-1}}{1-0.9997z^{-1}}$	$H(z)=\frac{0.2467x10^{-7}+0.4934x10^{-7}z^{-1}+0.2467x10^{-7}z^{-2}}{1-1.9996z^{-1}+0.9996z^{-2}}$
f_c=2.5 Hz	$H(z)=\frac{0.3925x10^{-3}+0.3925x10^{-3}z^{-1}}{1-0.9992z^{-1}}$	$H(z)=\frac{0.1541x10^{-6}+0.3083x10^{-6}z^{-1}+0.1541x10^{-6}z^{-2}}{1-1.9989z^{-1}+0.9989z^{-2}}$
f_c=5 Hz	$H(z)=\frac{0.7848x10^{-3}+0.7848x10^{-3}z^{-1}}{1-0.9984z^{-1}}$	$H(z)=\frac{0.0616x10^{-5}+0.1232x10^{-5}z^{-1}+0.0616x10^{-5}z^{-2}}{1-1.9978z^{-1}+0.9978z^{-2}}$
f_c=10 Hz	$H(z)=\frac{0.0016+0.0016z^{-1}}{1-0.9969z^{-1}}$	$H(z)=\frac{0.2462x10^{-5}+0.4924x10^{-5}z^{-1}+0.2462x10^{-5}z^{-2}}{1-1.9956z^{-1}+0.9956z^{-2}}$
	3rd order	
f_c=1 Hz	$H(z)=\frac{0.0387x10^{-10}+0.1162x10^{-10}z^{-1}+0.1162x10^{-10}z^{-2}+0.0387x10^{-10}z^{-3}}{1-2.9994z^{-1}+2.9987z^{-2}-0.9994z^{-3}}$	
f_c=2.5 Hz	$H(z)=\frac{0.0605x10^{-9}+0.1815x10^{-9}z^{-1}+0.1815x10^{-9}z^{-2}+0.0605x10^{-9}z^{-3}}{1-2.9984z^{-1}+2.9969z^{-2}-0.9984z^{-3}}$	
f_c=5 Hz	$H(z)=\frac{0.0484x10^{-8}+0.1451x10^{-8}z^{-1}+0.1451x10^{-8}z^{-2}+0.0484x10^{-8}z^{-3}}{1-2.9969z^{-1}+2.9937z^{-2}-0.9969z^{-3}}$	
f_c=10 Hz	$H(z)=\frac{0.0386x10^{-7}+0.1159x10^{-7}z^{-1}+0.1159x10^{-7}z^{-2}+0.0386x10^{-7}z^{-3}}{1-2.9937z^{-1}+2.9875z^{-2}-0.9937z^{-3}}$	

The frequency responses of the first-order Butterworth filters for various cut-off frequencies are given in Figure 3. Other filters with second and third order have similar frequency responses, and their amplitude and angle responses are not given here. In our problem, the desired output is the DC component and the gain of the designed filters appears to be 1 for 0 Hz (DC component). The amplitude response in Figure 3 shows that frequency components except the DC component in the filtered signal undergo rapid attenuation as frequency increases. The same figure shows that as the filter's cut-off frequency decreases, components except the DC component are more attenuated. It is also clear from the same figure that the filters produce nearly the same angle shift for all cut-off frequencies.

Figure 4 shows the frequency responses of three different filters with the same cut-off frequency (f_c = 5 Hz). The filters are first-, second-, and third-order low-pass Butterworth filters, respectively. These changes reveal that as the filter order increases, the attenuation of the high-frequency components also increases. This, as expected, indicates better filtering as the filter order increases.

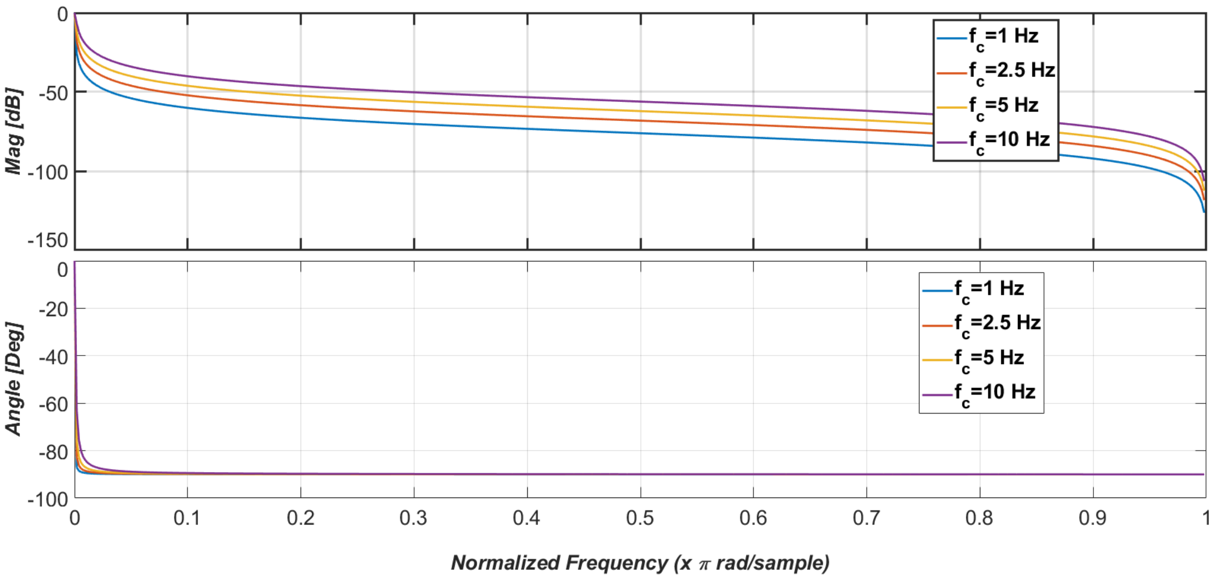

Figure 3. Frequency response of Butterworth filters for n=1 and f_c=1, 2.5, 5, 10 Hz

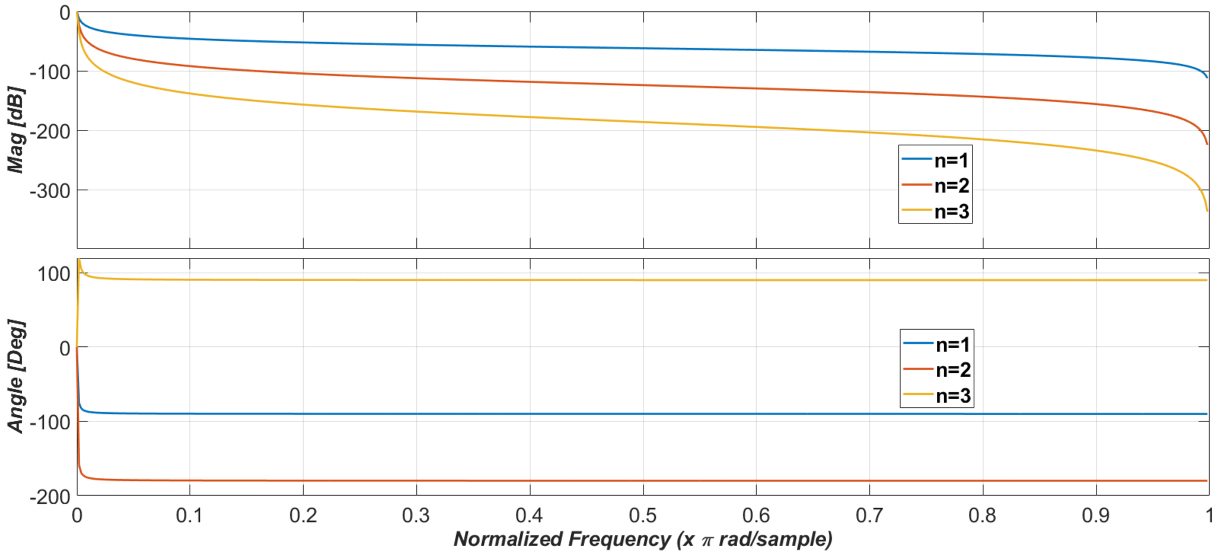

Figure 4. Frequency response of Butterworth filters for n=1, 2, 3 and f_c=5 Hz

Computed RMS values by designed Butterworth filters for various cut-off frequency (f_c=1, 2.5, 5 and 10 Hz) and orders (n=1, 2 and 3) are given Fig. 5 for voltage and Fig. 6 for current signal. If these figures are examined, it is generally observed that as the cut-off frequency of the filter decreases, the settling time increases but the oscillation at the filter output decreases. The filter order also affects the settling time and output oscillation. So much so that as the degree of the filter increases, the settling time also increases, but the oscillation is less and the smooth output is obtained. As a result of these observations, the cut-off frequency and order can be determined depending on the performance expected from the filter. It should be noted that as the filter degree increases, the computational load also increases. In general, the amount of high-frequency components in the filtered signal affects the performance of the filter. Since distortion in voltage is generally less than current, a smaller filter order can be designed for RMS computation of voltage. For example, a Butterworth filter with a cut-off frequency of f_c = 5 and an order of n = 3 is considered an appropriate RMS value estimator for voltage. Depending on the distortion in current, the RMS value estimator for current should be selected with a higher order.

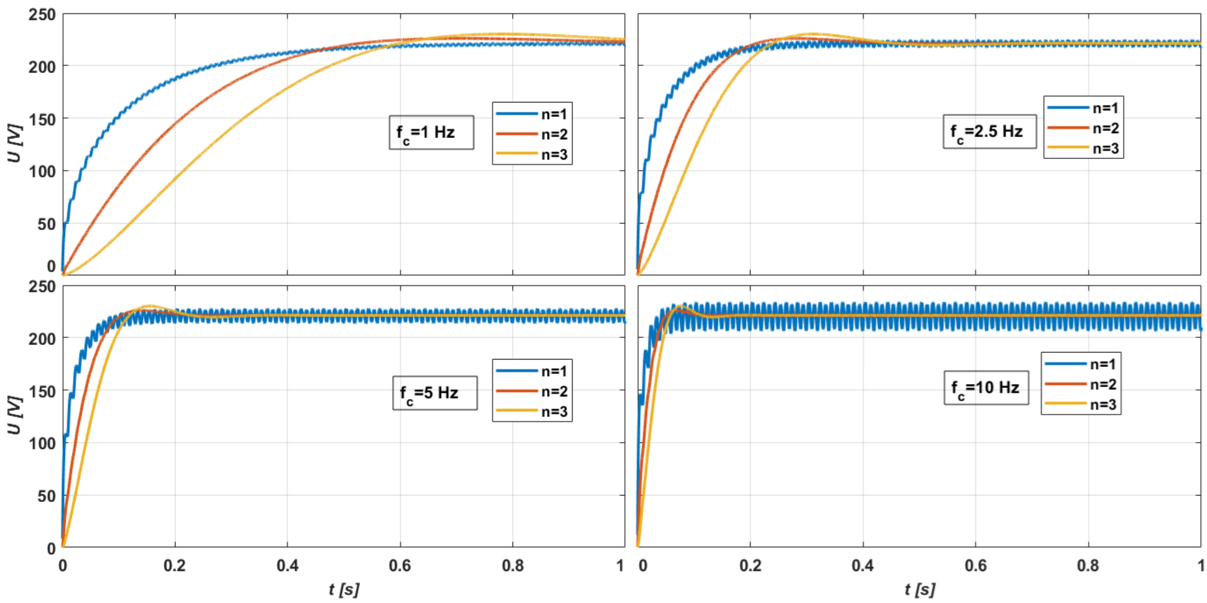

Figure 5. RMS values of the voltage obtained by Butterworth filters with various cut-off frequencies and orders

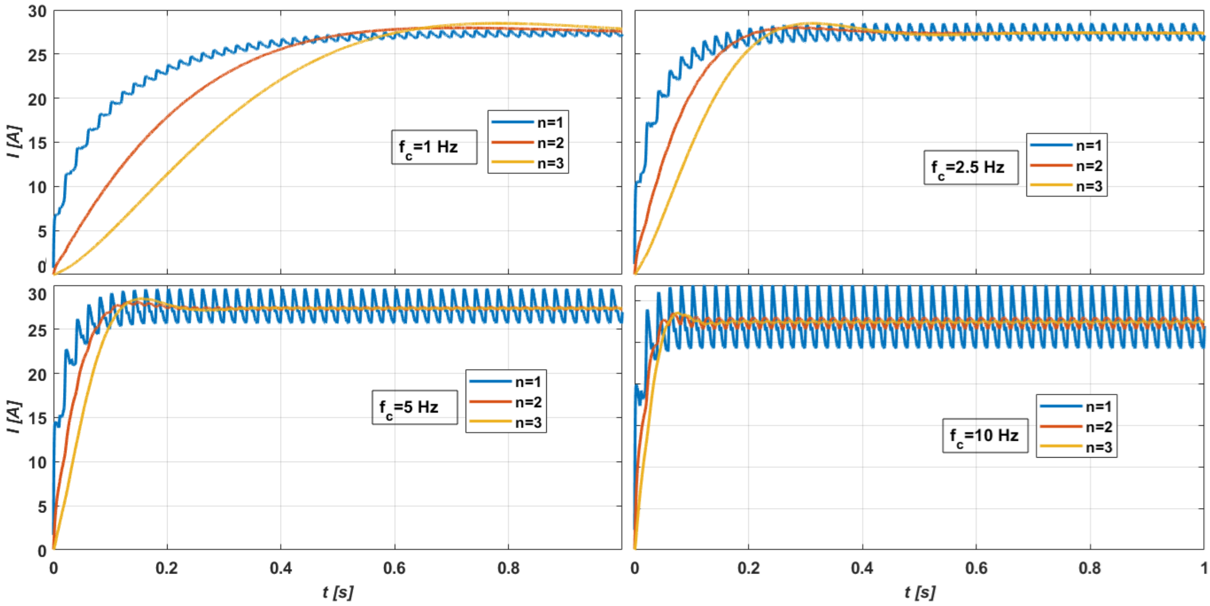

Figure 6. RMS values of the current obtained by Butterworth filters with various cut-off frequencies and orders

In order to test the filter's response to amplitude changes, the amplitude values of the voltage and current signals given by equations (21) and (22) were first increased by 25%, then restored to their original values and fed to the input of a third-order filter with a cut-off frequency of 5 Hz in a 2-second simulation. RMS values of the voltage and current obtained by the simulation, along with the actual values, are shown in Figure 7. This figure demonstrates that the designed filters can achieve new RMS values in a short time following changes in the voltage and current.

Similarly, in order to test the filters' response to frequency changes, the frequency values of the same voltage and current signals were first increased to 51 Hz, then restored to their original values and fed to the input of a third-order filter with a cut-off frequency of 5 Hz in a 2-second simulation. RMS values of the voltage and current obtained by the simulation, along with the actual values, are shown in Figure 8. The results in this figure demonstrate that changes in voltage or current frequency have virtually no impact on the filter performance. So that, although there is a very small deviation in the filter output after frequency changes at 0.8 s and 1.4 s, the real RMS value is reached shortly thereafter.

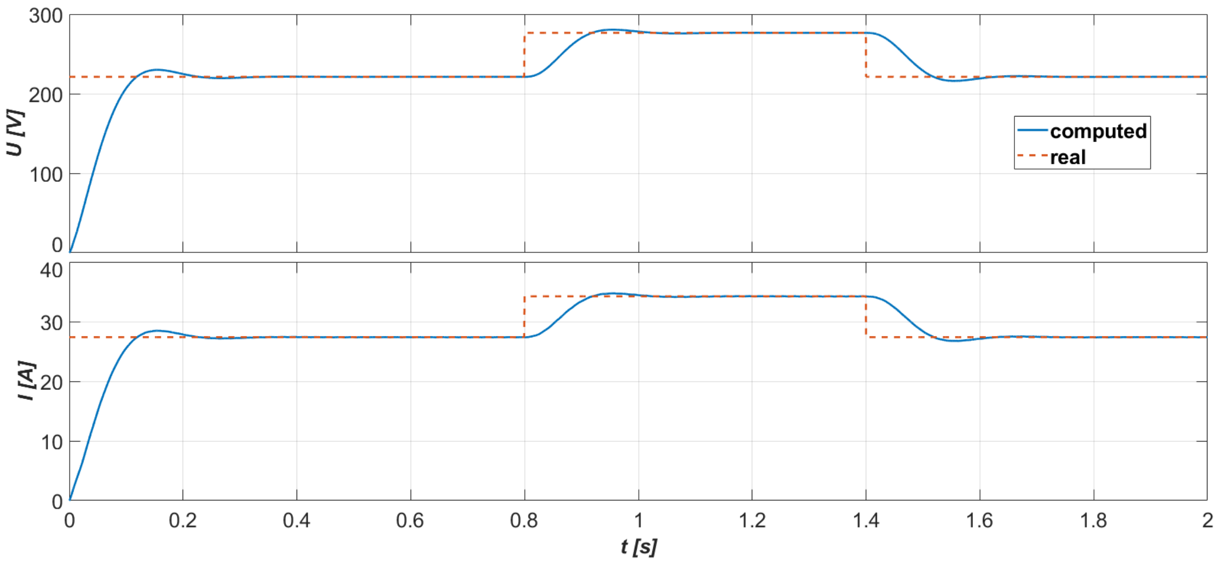

Figure 7. Real and Computed RMS values when the amplitude of voltage and current changes

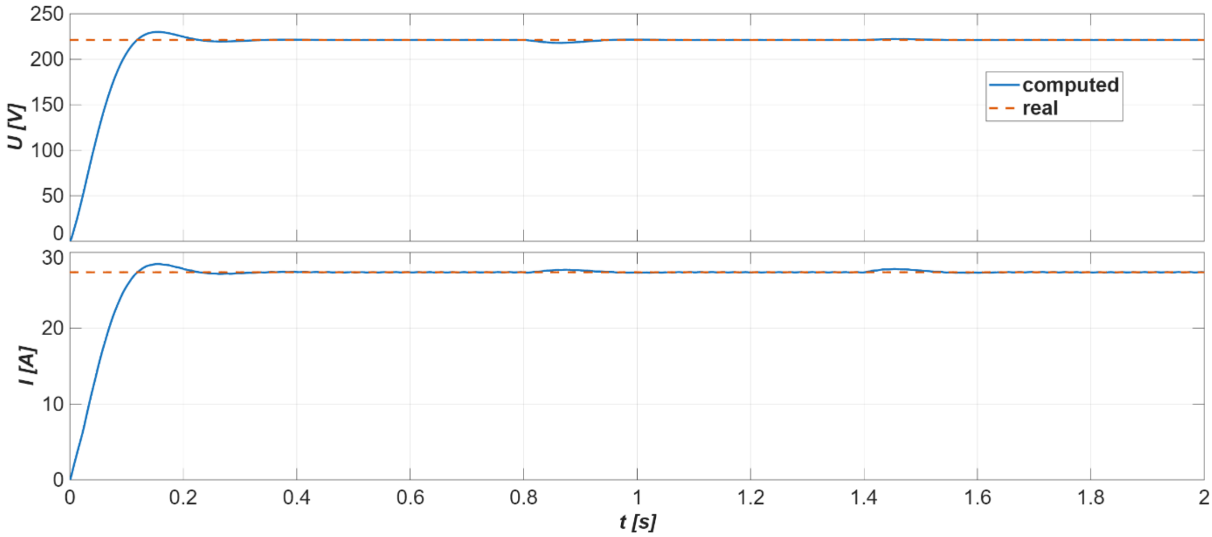

Figure 8. Real and Computed RMS values when the frequency of voltage and current changes

4. Conclusions

In this article, a filtering-based method has been introduced for measuring the RMS values of voltage and current of electrical power system. A standard filter structure, the Butterworth filter, was selected for filtering, and the effect of the filter's cut-off frequency and order on its performance was investigated through simulations done in MATLAB. The investigations showed that, in general, as the filter's cut-off frequency decreases, the settling time increases, but the oscillation at the filter output decreases. The filter degree also affects the settling time and output oscillation. As a result, as the filter order increases, the settling time also increases, but the oscillation is less, resulting in a smoother output. When the amplitudes and frequencies of the input signals of the designed filters change suddenly, a temporary drift occurs in the output signals. This drift is temporary, and true RMS values are achieved within a short time. In conclusion, this filtering-based method stands out as a highly advantageous method due to its low computational load, suitability for real-time calculations, and not required of frequency information for calculation.

In addition to the work presented in this article, comparisons can be made between standard filters in a future work. In addition, different types of filters can be compared in terms of computational load, filter order and noise immunity. Furthermore, a study on adaptive filter design for faster response could be conducted.

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