

Determination of Transmission Line Parameters by Utilizing Synchrophasor Measurements

Korhan Karaarslan*, Erdal Mustafa Yegin

Kocaeli University, Electrical Engineering, Kocaeli, Turkey

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Abstract

Modeling of transmission lines in electrical power systems is very important for power system engineering. Conventional methods for transmission line parameter determination take into account numerous factors. It is not applicable in most cases to use all the factors for parameter identification. PMU based methods, an innovative approach compared to the conventional methods, use synchrophasor voltage/current measurements from the both ends of the transmission line. In case of using PMU-based methods, transposition of transmission line gains importance in determining line parameters. Positive or negative sequence impedance parameters can be estimated accurately for a transposed transmission line. However, positive sequence model-based methods give rise to erroneous parameter estimations for an untransposed transmission line which is frequently encountered in practice. Therefore, three different PMU based methods are examined in case of transposed or untransposed line with balanced and unbalanced load.

Keywords: *transmission line parameter, synchrophasor measurement, PMU, estimation method, transposition*

1. Introduction

For a power transmission system, it is of great importance to know the transmission line (TL) parameters in order to reliably perform operations such as power flow, insulation coordination, transient stability analysis, protection coordination, state estimation, and etc.

There are various proposals of determination method in the past to estimate TL parameters. Although these parameters are available from the conductor data sheets of manufacturer, they vary depending on aging and operating conditions. Another approach is to measure TL parameters after construction based on Ohm's law. This measurement method can provide accurate values for the parameters but it is time consuming when considering all TLs [1,2]. There is a conventional estimation method based on Carson and Bessel functions that results in some errors due to approximations and assumptions that express the line physical characteristics (e.g. tower geometry, conductor dimensions, line height), environmental conditions and soil conductivity [3-6]. In both methods mentioned, it is not allowed for measuring when the line is powered.

Measurements of voltage and current taken from both the sending/receiving ends of the TL are used to estimate the line parameters. These measurements are called as synchrophasor acquired by using phasor measurement unit (PMU). From a theoretical point of view, synchrophasor measurement allows accurate estimation of TL parameters; however, from a practical perspective and based on methods in the literature, estimation of TL parameters is not easy owing to the fact that effectiveness of the estimation methods may vary depending on physical characteristics, system dynamics and load profile. Hence, each method can accurately estimate only a few parameters of TL under specific conditions and system dynamics.

To minimize estimation errors, various methods based on frequency-domain or time-domain are introduced [7-11]. In this paper, frequency-domain methods based on synchronized measurements obtained from PMUs are discussed. The widespread use of synchronized measurement-based methods to estimate TL parameters is limited by line transposition. In practice, transmission lines are usually untransposed. Therefore, applying the methods developed for transposed lines, will give rise

*Corresponding author: korhan.karaarslan@kocaeli.edu.tr

to erroneous parameter estimations in real world applications. This is due to mutually coupled sequence components for untransposed TLs. For transposed lines, three phase sequence circuits are fully decoupled and parameters of positive sequence lines are estimated by only positive sequence measurements [12-14]. This paper compares three methods to estimate TL parameters for transposed/untransposed TLs. In both cases, estimation methods use the nominal Π circuit model, shown in Fig. 1, to determine the TL parameters.

2. Estimation methods from synchronized measurements

There are three main types of transmission lines, classified according to line length as short, medium and long transmission lines. The nominal Π -circuit model with lumped parameters is used to represent the medium lines.

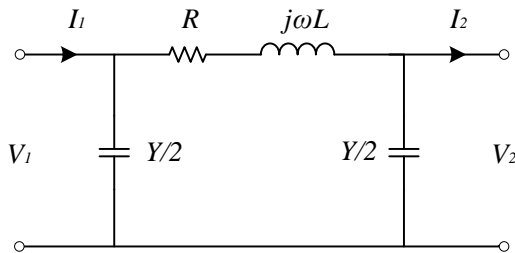


Fig.1 Nominal Π -circuit model for a transmission line

In this paper, a three-phase medium-length TL model is used. V_1, V_2, I_1 and I_2 denote respectively phasors at the sending/receiving ends for this model presented in Figure 1. Note that the impedance in series is in the middle of the circuit and the half of the shunt admittance is located at the both line ends.

Three-phase impedance matrix of a transposed TL has such a form that diagonal terms (self-impedances) are equal to each other. The same applies to off-diagonal terms (mutual impedances). Applying phase-to-sequence transformation to the impedance matrix, a symmetric diagonal matrix is obtained. This refer to three phase sequence circuits are completely de-coupled and positive sequence TL parameters are estimated only from positive sequence measurements.

2.1. Estimation methods for transposed TLs

Line parameter estimation methods from synchronized measurements utilize the nominal Π -circuit model for both transposed TL and untransposed TL. Among all the methods for transposed TLs, the single-measurement method is the simplest, as the name implies, only one set of synchrophasor measurement is sufficient [15]. By

applying Kirchhoff's laws for Figure 1, the following equations are obtained:

$$V_1 - V_2 = Z \left(\frac{1}{2} Y V_2 + I_2 \right) \tag{1}$$

$$I_1 - I_2 = \frac{1}{2} Y (V_1 + V_2) \tag{2}$$

The series impedance (Z) and shunt admittance (Y) are respectively defined as:

$$Z = R + jX \tag{3}$$

$$Y = jB_c \tag{4}$$

where X, R and B_c are the series reactance, series resistance and shunt susceptance of the TL. Rearranging equations (1) and (2) with the known quantities V_1, V_2, I_1 and I_2 measured by PMU, Z and Y can be obtained;

$$Y = 2 \left(\frac{I_1 - I_2}{V_1 + V_2} \right) \tag{5}$$

$$Z = \frac{V_1^2 - V_2^2}{V_1 I_2 + V_2 I_1} \tag{6}$$

Double-measurement method uses two sets of measurements from PMU to determine the ABCD parameters of a transposed line [16]. According to Figure 1, two-port ABCD parameters can be derived from following matrix:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{7}$$

Rearranging equation (7) for two sets of measurements yields equation (8) where the second subscript represents the number of phasor measurement sets.

$$\begin{bmatrix} V_{21} & I_{21} & 0 & 0 \\ 0 & 0 & V_{21} & I_{21} \\ V_{22} & I_{22} & 0 & 0 \\ 0 & 0 & V_{22} & I_{22} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} V_{11} \\ I_{11} \\ V_{12} \\ I_{12} \end{bmatrix} \tag{8}$$

Applying Cramer's Rule to the matrix in equation (8), ABCD parameters can be expressed as:

$$A = \frac{V_{12} I_{21} - V_{11} I_{22}}{\det} \tag{9}$$

$$B = \frac{V_{11}V_{22} - V_{21}V_{12}}{\det} \tag{10}$$

$$C = \frac{I_{12}I_{21} - I_{11}I_{22}}{\det} \tag{11}$$

$$D = \frac{V_{22}I_{11} - V_{21}I_{12}}{\det} \tag{12}$$

where $\det = V_{22}I_{21} - V_{21}I_{22}$.

Once the ABCD parameters are determined, the parameters of the transposed line can be obtained from the following equations:

$$A = 1 + \frac{1}{2}YZ \tag{13}$$

$$B = Z \tag{14}$$

$$C = Y \left(1 + \frac{1}{4}YZ \right) \tag{15}$$

$$D = 1 + \frac{1}{2}YZ \tag{16}$$

2.2. Estimation Method for Untransposed TLs

Estimation method presented can only be applied in some cases due to the fact that transmission lines are usually untransposed in practice [17]. The first optimal parameter estimation method for untransposed lines is presented by Di Shi, utilizes multiple measurements and least squares to determine TL parameters. It is well known that the sequence impedance matrix is not decoupled for untransposed lines.

Equations (1) and (2) obtained from nodal analysis are arranged for a three-phase nominal Π -circuit and rewritten as:

$$\bar{V}_1^{abc} - \bar{V}_2^{abc} = Z_{abc} \left(\frac{1}{2} Y_{abc} \bar{V}_2^{abc} + \bar{I}_2^{abc} \right) \tag{17}$$

$$\bar{I}_1^{abc} - \bar{I}_2^{abc} = \frac{1}{2} Y_{abc} \left(\bar{V}_2^{abc} + \bar{V}_1^{abc} \right) \tag{18}$$

These equations yield six complex equations. The equations obtained from Equation (17) are non-linear due to the multiplication of the unknown parameter matrices Z_{abc} and Y_{abc} by one other. Multiplying both sides by the inverse matrix of the phase impedance, Equation (17) becomes linear. The inverse Z_{abc} can be written as:

$$y_p = G_p + jT_p = Z_{abc}^{-1} = \begin{bmatrix} y_a & y_{ab} & y_{ac} \\ y_a & y_b & y_{bc} \\ y_{ac} & y_{bc} & y_c \end{bmatrix} \tag{19}$$

$(p = a, b, c, ab, ac, bc)$

Equation (17) and (18) can be expressed in matrix format as in Equation (20) and (21).

It is generally preferred to realize estimation by using least squares with real equations rather than complex equations. Equations (20) and (21) are expanded into 6 complex equations, which are then decomposed into 12 real expressions. To apply the least-squares method, a vector of measurement, a vector of parameter which is unknown and a matrix of coefficient are defined.

A measurement vector calculated from the PMU measurement can be defined as in Equation (22). The parameter vector which is unknown can be defined as in Equation (23).

$$\begin{bmatrix} y_a & y_{ab} & y_{ac} \\ y_a & y_b & y_{bc} \\ y_{ac} & y_{bc} & y_c \end{bmatrix} \begin{bmatrix} V_1^a - V_2^a \\ V_1^b - V_2^b \\ V_1^c - V_2^c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} Y_a & Y_{ab} & Y_{ac} \\ Y_a & Y_b & Y_{bc} \\ Y_{ac} & Y_{bc} & Y_c \end{bmatrix} \begin{bmatrix} V_2^a \\ V_2^b \\ V_2^c \end{bmatrix} + \begin{bmatrix} I_2^a \\ I_2^b \\ I_2^c \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} I_1^a - I_2^a \\ I_1^b - I_2^b \\ I_1^c - I_2^c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} Y_a & Y_{ab} & Y_{ac} \\ Y_a & Y_b & Y_{bc} \\ Y_{ac} & Y_{bc} & Y_c \end{bmatrix} \begin{bmatrix} V_1^a + V_2^a \\ V_1^b + V_2^b \\ V_1^c + V_2^c \end{bmatrix} \tag{21}$$

$$X = \begin{bmatrix} \text{Re}(I_2^a) & \text{Re}(I_2^b) & \text{Re}(I_2^c) & \text{Re}(\Delta I_a) & \text{Re}(\Delta I_b) & \text{Re}(\Delta I_c) & \dots \\ \dots & \text{Im}(I_2^a) & \text{Im}(I_2^b) & \text{Im}(I_2^c) & \text{Im}(\Delta I_a) & \text{Im}(\Delta I_b) & \text{Im}(\Delta I_c) \end{bmatrix}^T \tag{22}$$

$$\theta = \begin{bmatrix} G_a & T_a & G_b & T_b & G_c & T_c & G_{ab} & T_{ab} & G_{ac} & T_{ac} & G_{bc} & T_{bc} & \dots \\ \dots & B_a & B_b & B_c & B_{ab} & B_{ac} & B_{bc} \end{bmatrix}^T \tag{23}$$

The coefficient matrix, denoted as H, is derived from 12 real equations obtained from Equation (20) and (21) and contains measurements. The above definitions are rewritten in Equation (24) in a way to establish the least-squares problem.

$$X = H\theta \tag{24}$$

Three phase PMU samples are required for estimating the unknown parameters of a TL. Synchrophasor measurements taken at the same time instant is mentioned as one sample. Equation (24) describes the relationship among the values in a sample. For multiple PMU samples, Equation (24) becomes a set of overdetermined equations and least-square method can be applied for estimating the parameters of the TL. The parameter vector estimation can be expressed as in Equation (25).

$$\theta = (H^T H)^{-1} (H^T X) \tag{25}$$

Once θ is estimated, the sequence parameters of the phase impedance can be obtained from Equation (23) and using the sequence transformation.

3. Implementation of The Three Estimation Methods

Single-measurement method, double-measurement method and multiple measurement method using

least-squares are discussed through simulations, and the effectiveness of the three methods are compared. The TL modeled using the software EMTP has the characteristic parameters presented in Table I. The impedance parameters obtained from the simulation are considered as reference values for comparison. A 100 km-long untransposed TL with an unbalanced load is used to generate the PMU measurements. When single/double measurement methods are applied, the parameters of the untransposed line with unbalanced load are estimated inaccurately with significant errors. Table II shows the comparison of the actual values, obtained from the simulation as a reference, and estimation values.

Methods used to estimate the line parameters, should work correctly in both balanced and unbalanced load conditions. Since the load is unbalanced in real world applications, an unbalanced load is used for the simulation. Table III is created to compare the estimation results obtained from multiple measurement method using least-squares with simulation results. As it can be seen from the table, estimation of line parameters is achieved with a high accuracy. It should also be noted that the impedance matrix off-diagonal elements can be accurately calculated.

Table 1. Characteristic parameters of TL model for simulation

| Phase | Reactance (Ω/mile) | Conductor outer radius (inch) | Conductor resistance (Ω/mile) | Distance from the tower center (feet) | Vertical bundle height (feet) | Vertical bundle height at mid-span (feet) |
|-------|--------------------|-------------------------------|-------------------------------|---------------------------------------|-------------------------------|---|
| 1 | 0,31 | 0,75 | 0,0786 | 0 | 66 | 56 |
| 2 | 0,31 | 0,75 | 0,0786 | 0 | 84 | 74 |
| 3 | 0,31 | 0,75 | 0,0786 | 0 | 48 | 38 |

Table 2. Performance of single and double measurement methods when an untransposed TL is used

| Parameter | Single measurement method | | | Double measurement method | |
|----------------|---------------------------|------------------|-----------|---------------------------|-----------|
| | Actual values | Estimated values | Error (%) | Estimated values | Error (%) |
| $R_l (\Omega)$ | 0.70712 | 0.81020 | 14.57 | 1.00864 | 42.64 |
| $X_l (\Omega)$ | 5.53504 | 6.13208 | 10.78 | 6.12240 | 10.61 |
| $B_l (\Omega)$ | 4.01504e-05 | 3.69916e-5 | 07.86 | 3.69152e-05 | 08.06 |

Table 3. Performance of multiple measurement method with least-squares when an untransposed TL is used

| Parameter | Actual values | Multiple measurement method using Least-Squares | | |
|-------------------|------------------|---|----------------|----------------|
| | | Estimated values | Error (%) in R | Error (%) in X |
| $Z_l (\Omega)^*$ | 0.70712+j5.53504 | 0.70924+j5.55773 | 3.3e-03 | 4.1e-03 |
| $Z_o (\Omega)$ | 2.64640+j18.2445 | 2.64716+j18.3685 | 2.9e-03 | 6.8e-03 |
| $Z_{l0} (\Omega)$ | -0.16432-j0.1044 | -0.16515-j0.10518 | 51e-03 | 75e-03 |
| $Z_{l2} (\Omega)$ | - | -0.35043+j0.20187 | 26e-03 | 2.2e-3 |
| | 0.34952+j0.20192 | | | |
| $Z_{20} (\Omega)$ | 0.17704-j0.11168 | 0.17701-j0.111671 | 1.3e-03 | 1.0e-03 |
| $Z_{21} (\Omega)$ | 0.34968+j0.20176 | 0.34979+j0.20170 | 3.3e-03 | 2.6e-03 |
| $Z_{01} (\Omega)$ | 0.17704-j0.11168 | 0.17701-j0.111671 | 1.3e-03 | 1.0e-03 |
| $Z_{02} (\Omega)$ | -0.16432-j0.1044 | -0.16515-j0.10518 | 51e-03 | 75e-03 |
| $B_l (S)$ | 4.01504e-05 | 4.01431e-05 | - | 18e-03 |
| $B_o (S)$ | 2.2104e-05 | 2.20863e-05 | - | 8e-03 |
| $B_{01} (S)^*$ | 1.2644e-06 | 1.26454 | - | 11e-03 |
| $B_{02} (S)$ | 1.2644e-06 | 1.26441 | - | 1.1e-03 |
| $B_{l2} (S)^*$ | -1.34952e-06 | -1.34985 | - | 2.5e-03 |

* $B_1 = B_2$, $B_{02} = B_{20}$, $B_{01} = B_{10}$, $B_{l2} = B_{21}$, $Z_1 = Z_2$

4. Conclusions

Three well-known TL parameter estimation methods are compared in this paper. A medium-length untransposed transmission line is used for simulation model and obtaining synchrophasor measurements, because transmission lines are usually untransposed in practice. Simulations are also conducted in unbalanced load conditions as it is more realistic.

Single/double measurement methods differ from multiple measurement method using least-squares. The main reason for this is that these two methods are

based on positive-sequence circuit model of a transmission line. Therefore, performance of single/double measurement methods is poor in particular when TL is not transposed and unbalanced load condition exists.

Unlike the methods based on positive-sequence circuit model, multiple measurement method using linear least-squares performs significantly better estimations for both transposed and untransposed TLs under balanced and unbalanced loadings.

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