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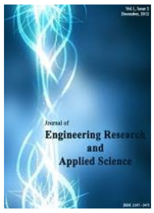

Optimization of the tank position and aspect ratio by the Box-Behnken method for enhancement of thermal stratification in a hot water tank

A. Yıldız[1], B. Kurşun[2], K. Ökten[2]

[1] Bursa Technical University, Industrial Engineering, Bursa, Turkey
[2] Amasya University, Mechanical Engineering, Amasya, Turkey

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Abstract
Thermal stratification in hot water storage systems ensures that the hot water temperature in the tank can be maintained for a longer time and the energy of the heat source can be utilized more efficiently. In the literature, many experimental and numerical studies have been carried out to increase the thermal stratification by changing the tank design. In this study, however, it was aimed to increase the thermal stratification in the tank by only changing the position of the tank. The effects of the angle of the tank with the horizontal axis (α) and the tank aspect ratio (D/H) on thermal stratification were investigated numerically. Also, an optimization was carried out with the Box-Behnken method to determine the most suitable aspect ratio and tank position in terms of thermal stratification and second law efficiency. As a result of the optimization process, it was determined that the most suitable α angle was 47.23˚ and the D/H ratio was 0.922. Depending on these parameter values, the temperature difference and second law efficiency in the tank were obtained as 11,651˚ and 0.898, respectively. The results of the analysis revealed that the applied method can contribute to the formation of thermal stratification in hot water tanks.

Keywords: Box-benkhen, hot water tank, thermal stratification.

1. Introduction

The increasing use of fossil fuels around the world pushes countries to use renewable energy sources and to use existing energy efficiently [1,2]. Therefore, both the storage of waste heat and the storage of the hot source obtained from renewable energy sources are important. Heat storage from hot source; It can be collected under 3 main headings as hidden, sensible and thermochemical storage. However, sensible heat storage comes to the forefront due to its low cost and ease of use. Especially hot water tanks are very common in heating and cooling applications because they are non-toxic, non-corrosive and have low vapor pressure. Three zones are formed in the hot water storage tank due to convection effects. There is a hot water zone at the top of the tank, a cold-water zone at the bottom, and a mixing zone where hot and cold water meet in the middle [3].

It is a known fact that increasing the thermal stratification in the tank increases the storage efficiency [4]. There are many studies in the literature to increase thermal stratification for static and dynamic situations. However, since the static situation was examined in this study, the literature studies conducted under the static situation were examined. Olevski et al. [5] investigated the thermal stratification effect experimentally and numerically depending on the different aspect ratios and storage times of the thermal storage tank. Altuntop et al. [6] placed fins with 12 different geometries in the middle of the heat storage tank and examined which geometry increased the thermal stratification the most. In another study, Erdemir and Altuntop [7] investigated thermal stratification by placing wings at different heights in the tank. Lin and Armfield [8] investigated thermal stratification in cylindrical and rectangular thermal storage tanks.

A more comprehensive study was carried out by Yang et al [9]. In the same volume, the effect of 7 different tank geometries on thermal stratification was investigated numerically. As a result of the study, it was determined that the tanks with sharp corners increased the thermal stratification. Apart from changing the tank geometry and placing fins inside the tank, Kurşun [10] increased the thermal stratification by placing conical shaped insulation

[a] Corresponding author; aytac.yildiz@btu.edu.tr

outside the thermal storage tank.

Considering the literature studies reviewed above, the presence of sharp corners in the tank, tank aspect ratio and storage time are among the most important parameters affecting thermal stratification. Especially in the study conducted by Kurşun and Ökten [11], rectangular thermal storage tanks were examined at different angles of inclination to increase the formation of sharp corners in the tank, and at different aspect ratios and storage times to examine the thermal stratification. The study was performed for 0˚, 45˚, and 60˚ tilt angles, 0.5, 0.7, and 1 aspect ratios, and storage times of 2, 4, 6, and 8 hours. As a result of the study, thermal storage tank, 1st law, 2nd law efficiencies and temperature differences were examined for each case. However, the optimum storage time, tilt angle and aspect ratios have not been determined depending on the inputs. Therefore, in this study, thermal stratification was investigated depending on the inclination angle of the thermal storage tank, aspect ratio and storage times and optimum values were determined.

For the optimization, Box-Behnken factorial experimental design combined with response surface methodology was made and optimum values were determined depending on all the inputs.

2. Numerical study

Fig. 1 shows the rectangular tank geometry used for numerical analysis. Thermal stratification in the tank was investigated by performing numerical analysis based on time for different aspect ratios (D/H) of the tank and the different angles (α) of the bottom surface of the tank with the horizontal.

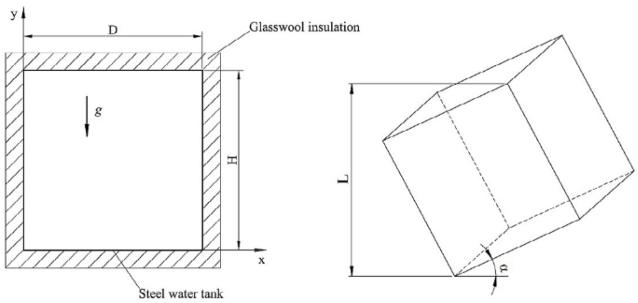


Figure 1. Rectangular tank geometry and sectional view

Conservation equations used for numerical analysis are solved by discretization in a two-dimensional time-dependent manner. The finite volume method was used for numerical analysis. SIMPLE algorithm was preferred for solving pressure-velocity couple together and PRESTO method was preferred for discretization of pressure equations. Reference [11] can be examined for details of the mathematical model used and the cell structure in the region where the numerical analysis is performed. Temperature-dependent numerical results obtained by solving the created mathematical model were compared with the experimental results in reference [9]. As a result of the comparison, it was observed that the highest difference between the experimental and numerical study results was 1.5%. Detailed information on the validation of the numerical study can be obtained by examining the reference [11].

3. Material and method

In this study, a three-level Box-Behnken factorial experimental design combined with the response surface methodology was used to optimize the "tank position, aspect ratio and time" factors used for maximum temperature difference and second law efficiency in optimum tank design. This experimental design method is briefly summarized below.

3.1. Response surface method

Response surface method (RSM) is a statistical and mathematical method in which desired responses are expressed as a function of input variables [12, 13, 14]. The main purpose of this method is to optimize the response surface, which is affected by various process factors. The basic principle of RSM is to obtain an optimal response through a well-prepared series of experiments. Statistical techniques such as RSM can be used to optimize operational variables to maximize the production of a particular substance. Using appropriate experiment design (DoE), RSM has recently become popular for formulation optimization. The relationship between the controlled input parameters and the response surfaces created can also be determined using the response surface technique [12, 15,16,17].

If the response of the system in RSM gives a good fit as a linear function of the independent variable, the following first-order model is obtained from the method [18].

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon \quad (1)$$

If the response surface of the system has a curvature, a quadratic model may be more appropriate.

$$y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=1}^{k} \beta_{jj} x^2{}_j + \sum_{i}^{k-1} \sum_{j}^{k} \beta_{ij}\, x_i x_j + \varepsilon \quad (2)$$

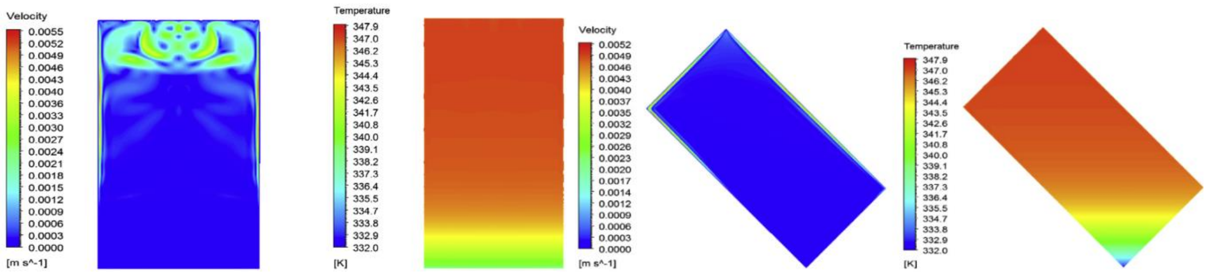
In this equation; y response variable, β_0, β_1, β_2,…. β_k unknown regression parameters, x_i,x_j process(decision) variables $(i=1, 2, \ldots, k)$ and $(j=1, 2, \ldots, k)$, and ε error term [18].

There are two types of design in the RSM method, namely Central Composite Design (CCD) and Box-Behnken Design (BBD) [12, 16].

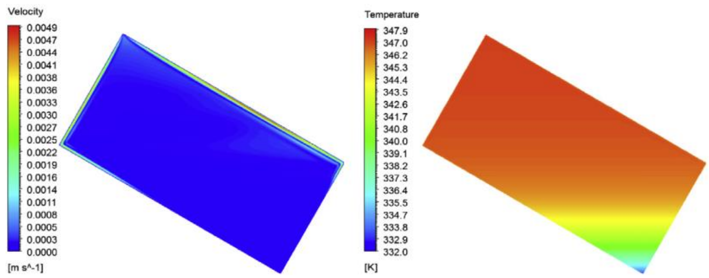
Box-Behnken designs are a response surface design that is used because it allows the estimation of quadratic models, the establishment of sequential designs, the analysis of the model's lack of confidence, and blocks [19] and is effectively used in experimental designs. Because the use of this method in experimental design models helps the experiment to be done quickly and greatly reduces the testing costs [20].

4. Findings and discussion

Numerical analyses carried out the rectangular tank with different aspect ratios (D/H=1, 0.7, 0.5), different angles with the horizontal axis (α=0˚, 45˚, 60˚) and different cooling periods (t=2, 4, 6 and 8 hours). Fig. 2 shows the temperature and velocity contours obtained at different α angles for a certain cooling period. It is seen that the natural convection movement that occurs as a result of cooling decreases with the change of the tank position (α=45˚and 60˚). It is seen that the velocity boundary layer formed on the tank walls in the vertical position of the tank reaches the bottom surface of the tank. This situation has led to an increase in the natural convection movement that occurs due to the density difference between hot and cold water. Thus, a mixture occurred in the tank and this mixing action caused them to decrease the temperature of water in the hot zone. In other tank positions, the velocity boundary layer formed on the tank walls could not reach the bottom of the tank. Thus, the mixing of hot and cold water is prevented to a certain extent and thermal stratification is increased.



a) b)



c)

Figure 2. Temperature and speed counters D/H=0.5, a) α=0˚, b) α=45˚, c) α=60˚

The graphs in Fig. 3 show the second law efficiencies calculated depending on the average water temperature in the tank, which changes according to the α angle and the D/H ratio as a result of cooling [11].

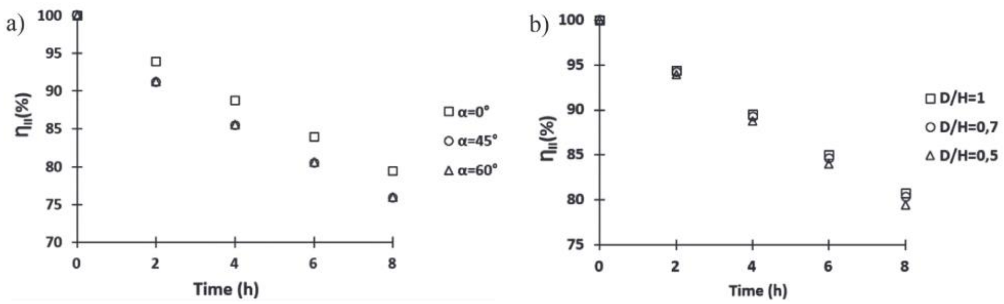


Figure 3. Second law-efficiencies for the 8 hour cooling period, a) D/H=0.5, b) α=45˚

While the change in the D/H ratio affected the second law efficiency at a negligible level, a certain decrease occurred in the second law efficiency depending on the α angle. As it can be understood from the numerical study results, the change of D/H ratio and α angle affects both the thermal stratification and the second law efficiency at different rates depending on time. Therefore, optimum D/H ratio and α angle values need to be determined. For optimum values, Box-Behnken experimental design was carried out using the numerical results obtained.

After the experimental design to be used in the study was determined, the factors affecting the tank design and their levels were determined and given in Table 1.

Table 1. Factors affecting tank design and their levels

Factors	Symbol	Level		
		Low -1	**Center 0**	**High +1**
Tank Position (degree)	*T*	0	45	60
Aspect Ratio	*A*	0.5	0.7	1
Time (hour)	*t*	2	4	6

According to the Box-Behnken experimental design, the number of experiments to be performed was determined as 17 experiments. Numerical analyses were made in accordance with the experimental set created and the obtained "temperature difference" and "second law efficiency" values are given in Table 2. The values given in parentheses in Table 2 show the 3-level structure used in the Box-Behnken experimental design, while the other values show the actual values corresponding to these levels.

Table 2. Box–Behnken experimental design and numerical analysis results

Run no	**Actual and coded level of variables**			**Numerical analysis response**	
	Tank Position (degree)	Aspect Ratio	Time (hour)	Temperature Difference (˚C)	Second Law Efficiency (SLE)
1	45 (0)	1 (1)	2 (-1)	10.265	0.912

2	0 (-1)	0.7 (0)	2 (-1)	4.699	0.9425
3	45 (0)	0.5 (-1)	2 (-1)	10.444	0.8192
4	60 (1)	0.7 (0)	2 (-1)	10.204	0.9142
5	60 (1)	0.5 (-1)	4 (0)	12.25	0.8559
6	45 (0)	0.7 (0)	4 (0)	12.694	0.8575
7	45 (0)	1 (1)	6 (1)	13.575	0.809
8	45 (0)	0.7 (0)	4 (0)	12.694	0.8575
9	60 (1)	1 (1)	4 (0)	12.139	0.8605
10	45 (0)	0.7 (0)	4 (0)	12.694	0.8575
11	0 (-1)	0.5 (-1)	4 (0)	6.422	0.8878
12	45 (0)	0.5 (-1)	6 (1)	13.579	0.8049
13	0 (-1)	1 (1)	4 (0)	5.743	0.8952
14	60 (1)	0.7 (0)	6 (1)	13.308	0.8104
15	45 (0)	0.70 (0)	4 (0)	12.694	0.8575
16	0 (-1)	0.70 (0)	6 (1)	6.917	0.8467
17	45 (0)	0.70 (0)	4 (0)	12.694	0.8575

For the numerical analysis responses obtained, analyzes were made using the Design Expert 11 package program, and a quadratic regression model suitable for the optimum tank design factors giving the maximum temperature difference and second law efficiency was created as in equations (1) and (2).

$$R_{TD}=+12.69+3.02T-0.1216A+1.47t+0.142TA+0{,}222Tt+0{,}044At-3.37T^2-0.1859A^2-0.542t^2 \quad (1)$$

$$R_{SLE}=+0.857-0.016T+0.014A-0.039t-0.0007TA-0.002Tt-0.022At+0.029T^2-0.0124A^2-0.0088t^2 \quad (2)$$

Analysis of variance (ANOVA) was performed to determine the contribution of the factors affecting the tank design to the "temperature difference" and "second law efficiency" parameters, and the quadratic model analysis of variance results are given in Table 3 and Table 4.

Table 3. ANOVA analysis for temperature difference

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	141.12	9	15.68	505.45	< 0.0001	significant
T	72.72	1	72.72	2344.30	< 0.0001	significant
A	0.1183	1	0.1183	3.81	0.0917	
t	17.31	1	17.31	557.94	< 0.0001	significant
TA	0.0807	1	0.0807	2.60	0.1509	
Tt	0.1962	1	0.1962	6.33	0.0401	significant
At	0.0077	1	0.0077	0.2468	0.6346	
T^2	47.81	1	47.81	1541.16	< 0.0001	significant
A^2	0.1455	1	0.1455	4.69	0.0671	
t^2	1.24	1	1.24	39.93	0.0004	significant
Residual	0.2171	7	0.0310			
Total	141.33	16				

Adequate precision = 66,6024
R^2 = 0.9985, R^2 (predicted) = 0.9754, R^2 (adjusted) = 0.9965

In Table 3, it was determined that the model was significant (P-values <0.05) in the analysis performed for the temperature difference parameter. In this analysis, it is seen that *T, t, Tt,* T^2, t^2 model factors are significant and therefore they are effective on the "temperature difference" parameter. Considering the significance values, it was determined that time *(t)* was the most effective factor on the "temperature difference".

Adjusted R^2 (0.9965) indicates that 99.65% of the variance in the response can be explained by the quadratic model. For adequate precision, a ratio greater than 4 (66,6024) is desirable.

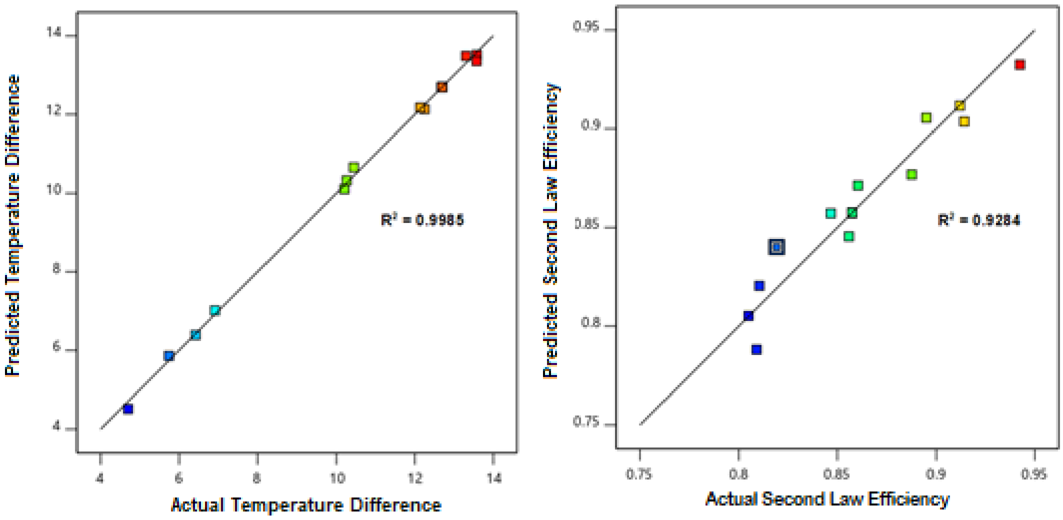
Table 4. ANOVA analysis for second law efficiency

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.0227	9	0.0025	10.08	0.0030	significant
T	0.0022	1	0.0022	8.62	0.0219	significant
A	0.0015	1	0.0015	5.94	0.0450	significant
t	0.0126	1	0.0126	50.27	0.0002	significant
TA	1.960E-06	1	1.960E-06	0.0078	0.9319	
Tt	0.0000	1	0.0000	0.0641	0.8075	
At	0.0020	1	0.0020	7.88	0.0263	significant
T^2	0.0037	1	0.0037	14.93	0.0062	significant
A^2	0.0006	1	0.0006	2.60	0.1511	
t^2	0.0003	1	0.0003	1.31	0.2901	
Residual	0.0017	7	0.0002			
Total	0.0244	16				

Adequate precision = 66,6024
R^2 = 0.9284, R^2 (predicted) =-0.1460, R^2 (adjusted) = 0.8363

According to Table 4, the model made for the "second law efficiency" parameter was found to be significant (P-values <0.05). In this analysis, it is seen that *T, A, t, At,* T^2 model factors are significant and therefore they are effective on the "second law efficiency" parameter. Considering the significance values of the factors, it is seen that time *(t)* is the most effective factor on "second law efficiency" as in the temperature difference parameter. Then, it was determined that the tank position and aspect ratio factors were effective on efficiency, respectively. Adjusted R^2 (0.8363) indicates that 83,63% of the variance in the response can be explained by the quadratic model. A negative Predicted R^2 implies that the overall mean may be a better predictor of your response than the current model. In some cases, a higher order model may also predict better.

Estimated values obtained from numerical analysis and quadratic regression model equations for temperature difference and second law efficiency parameters are given in Fig. 4.



(a) (b)

Figure 4. Box-Behnken versus experimental predicted (a) temperature difference and (b) second law efficiency

It is seen in Fig. 4 that the estimated values are compatible with the actual numerical analysis results. This result shows that the developed quadratic regression model can be used to optimize the factors that are effective for the tank design.

After the ANOVA analysis, the effects of the factors affecting the tank design on the "temperature difference" and "second law efficiency" parameters were examined and the findings are given in Fig. 5 and Fig. 6.

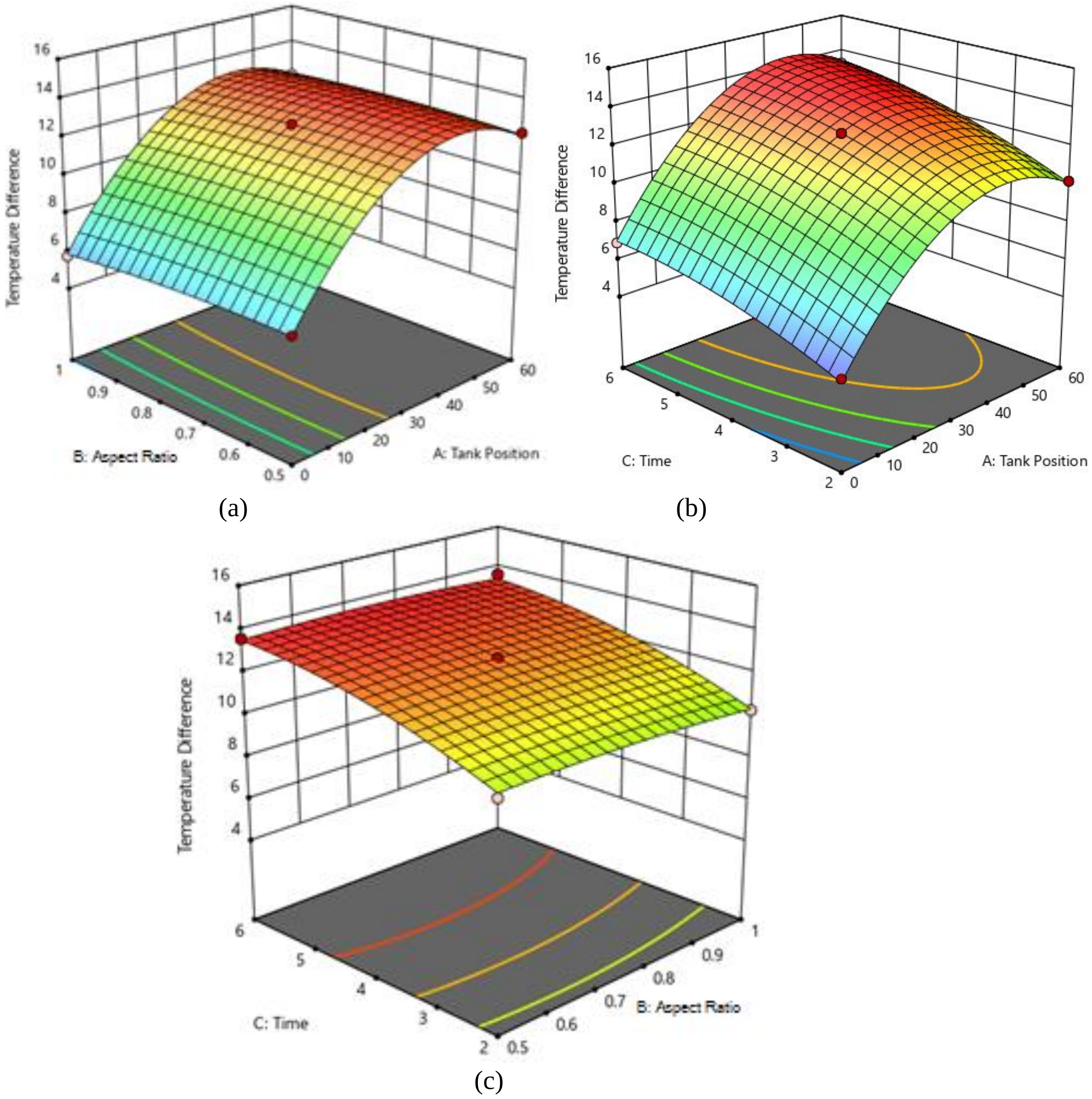


Figure 5. The effect on the temperature difference of the factors affecting the tank design

Fig. 5 shows the effect of the factors affecting the tank design on the temperature difference. Accordingly, when Fig. 5(a) is examined, it is seen that the angle of the tank position has a great effect on the temperature difference, but the tank aspect ratio does not have a significant effect. At the same time, when the angle of the tank position increases up to a certain degree (47,33°), the temperature difference also increases, but after this degree the temperature difference also decreases. In Fig. 5(b), it is seen that both the tank position and time factors have a significant effect on the temperature difference parameter. As time increases, the temperature difference also increases. The effect of tank aspect ratio and time factors on the temperature difference is given in Fig. 5(c). Accordingly, it is seen that the temperature difference increases as time increases, but the aspect ratio does not have a significant effect on the temperature difference.

In Fig. 6, the effects on the second law efficiency of the factors affecting the tank design are given.

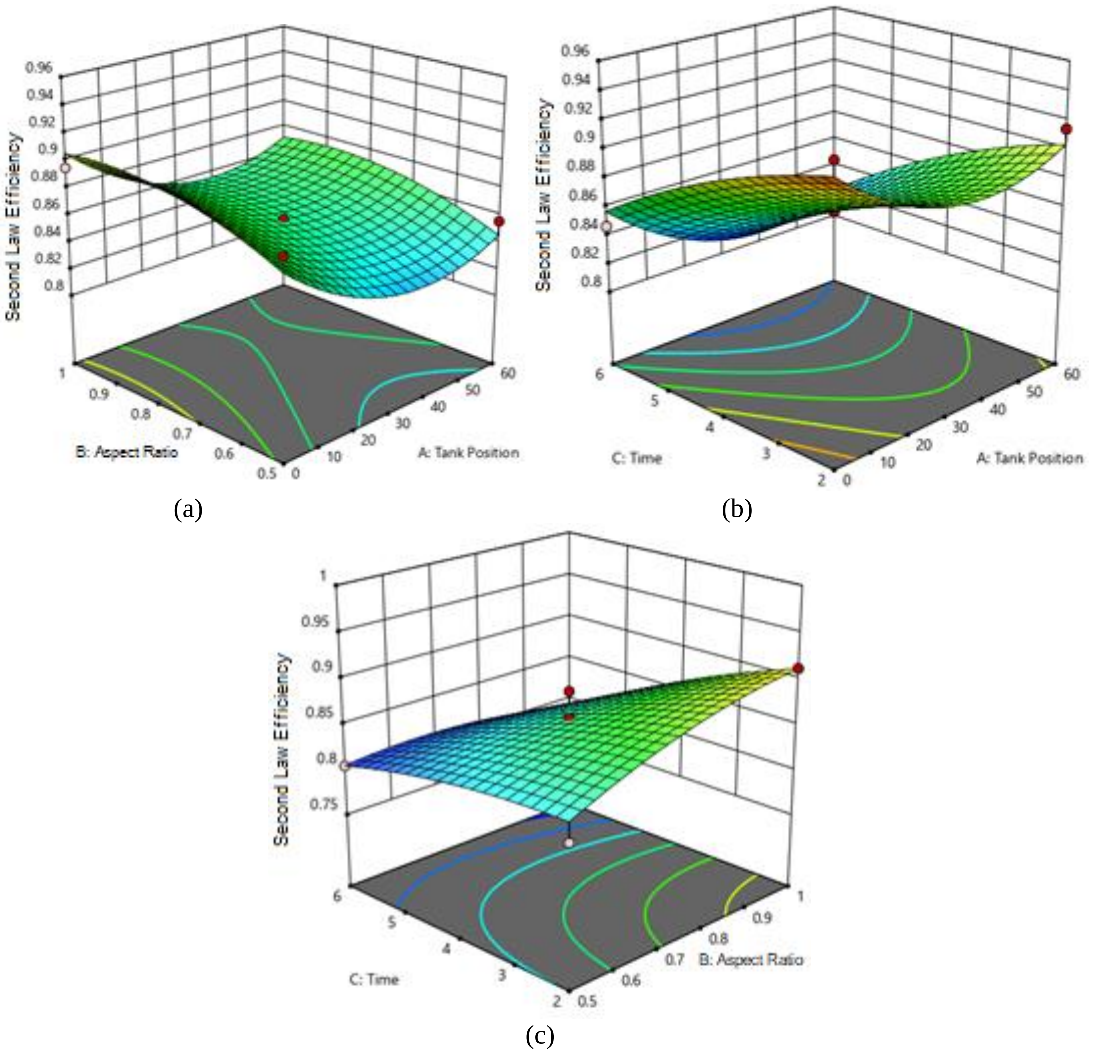


Figure 6. The effect on second law efficiency of the factors affecting the tank design

When Fig. 6(a) is examined, it is seen that the second law efficiency is high when the tank position angle is low, but the efficiency decreases as the angle increases, and the second law efficiency increases again after about 30˚. When the aspect ratio is examined, it is seen that the second law efficiency increases with the increase in the ratio. In Fig. 6(b), it is seen that the second law efficiency decreases with increasing time. It can be seen in Fig. 6(c) that the second law efficiency increases with increasing aspect ratio, whereas it decreases with increasing time. After examining the effects of the factors affecting the tank design on the temperature difference and second law efficiency, the optimum tank design factors value for the highest temperature difference and second law efficiency were obtained from the Design Expert 11 program. The optimum tank design factors values obtained and the corresponding temperature difference and second law efficiency values are given in Table 5.

Table 5. Optimum tank design factors and maximum temperature difference-second law efficiency values

Tank Position(degree)	Aspect Ratio	Time (hour)	Temperature Difference (˚C)	Second Law Efficiency	Desirability
47.33	0.922	2.485	11.651	0.898	0.677

Numerical analysis was made according to these optimum tank design factors values (*D/H=0.922, α=47.33˚* and *t=2.485* hours) and as a result of the analyses made, the "temperature difference" and "second law efficiency" values were obtained as 11,905˚ and 0,92 respectively.

In this case, it can be said that the Box-Behnken experimental design and the numerical study give similar results (with only a 2% margin of error).

5. Conclusion and evaluation

In this study, it is aimed to optimize the tank design factors affecting the temperature difference and second law efficiency parameters with the three-level Box-Behnken design combined with the response surface method. For this, the temperature difference and second law efficiency values obtained from the numerical analysis based on the tank position angle, aspect ratio and time factors were analysed in the Design Expert 11 program. Tank design factors affecting temperature difference and second law efficiency parameters were optimized. The results obtained from the study are listed below.

- In the experiment result estimates made with Box-Behnken factorial design, R^2 (adjusted) = 0.9965 for "temperature difference" and R^2 (adjusted) = 0.8363 for "second law efficiency", and this optimization design can make accurate predictions for this problem.
- According to the factors and levels used in the analysis; It has been determined that the tank position and time factors are important for the temperature difference parameter and the most important factor is the tank position, and the factor that affects the second law efficiency parameter the most is time.
- The factor that has the least effect on both the temperature difference and the second law productivity parameters is the aspect ratio.
- For maximum temperature difference and second law efficiency, the tank position angle should be 47.33 degrees, the aspect ratio should be 0.922 and the time should be 2.485 hours.
- Box-Behnken design can be used for tank design because the maximum temperature difference and second law efficiency values obtained by numerical analysis and Box-Behnken design are very close to each other.

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